Graph II

CS 70 Discussion 2B

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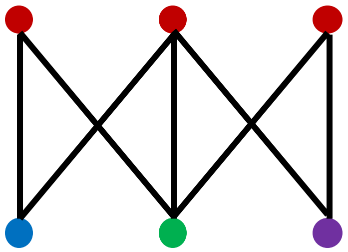
Problem 1: Always, Sometimes, Never

a) G can be vertex-colored with 4 colors.

- Planar graphs: True, by four/five color theorem
- Nonplanar graphs: True, check $K_{3,3}$ or K_5

Either



b) G requires 7 colors to be vertex-colored

- Planar graphs: False, only need four/five color
- Nonplanar graphs: True

Always Nonplanar

Problem 1: Always, Sometimes, Never

c) $e \leq 3v - 6$

- Planar graphs: True, by Euler formula
- Nonplanar graphs: True, check $K_{3,3}$ or K_5

<u>Either</u>

d) G connected, each vertex has degree at least 2

- Planar graphs: True
- Nonplanar graphs: False, by Kurwatowski's theorem

<u>Always planar</u>

e) Every vertex has degree at least 2

- Planar graphs: True
- Nonplanar graphs: False, by Kurwatowski's theorem

<u>Always planar</u>

Problem 2: Short Answers

a) A connected planar simple graph has 5 more edges than it has vertices. How many faces does it have?

Goal: Want to get a bound on f

$$v - e + f = 2$$

Condition: $e = 5 + v$

Plug in:

$$v - (5 + v) + f = 2$$

$$f = 7$$

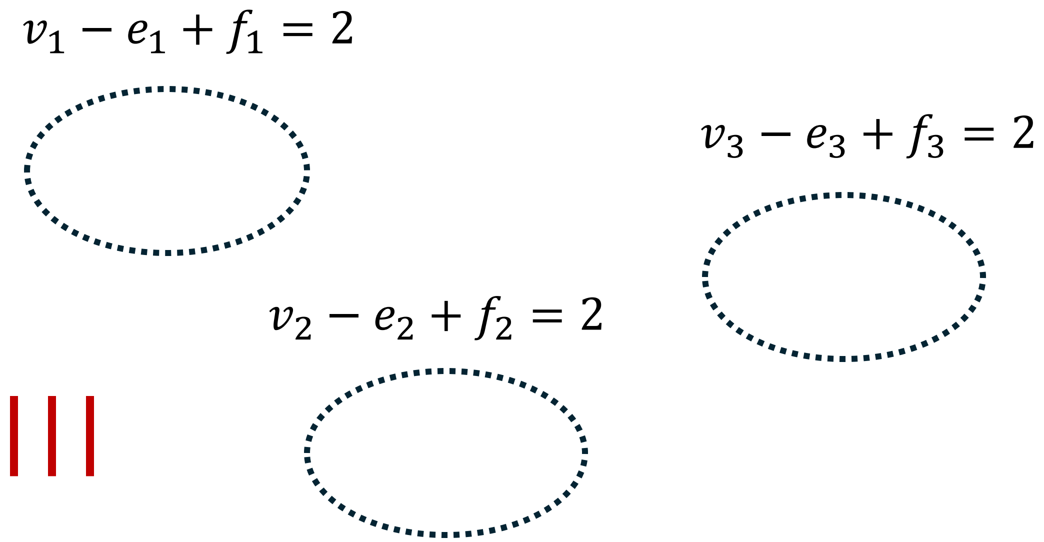
Problem 2: Short Answers

b) How many edges need to be removed from a 3-dimensional hypercube to get a tree?

- We now that 3-dimensional hypercude have 8 vertices
- Trees on 8 vertices have 7 edges
- A 3-hypercube has 12 edges
- Need to remove 5 edges

Problem 2: Short Answers

c) The Euler's formula $v - e + f = 2$ requires the planar graph to be connected. What is the analogous formula for planar graphs with k connected components?



Assume there are 3 connected components

$$\sum_{i=1}^{k} (v_i - e_i + f_i) = \sum_{i=1}^{k} 2$$

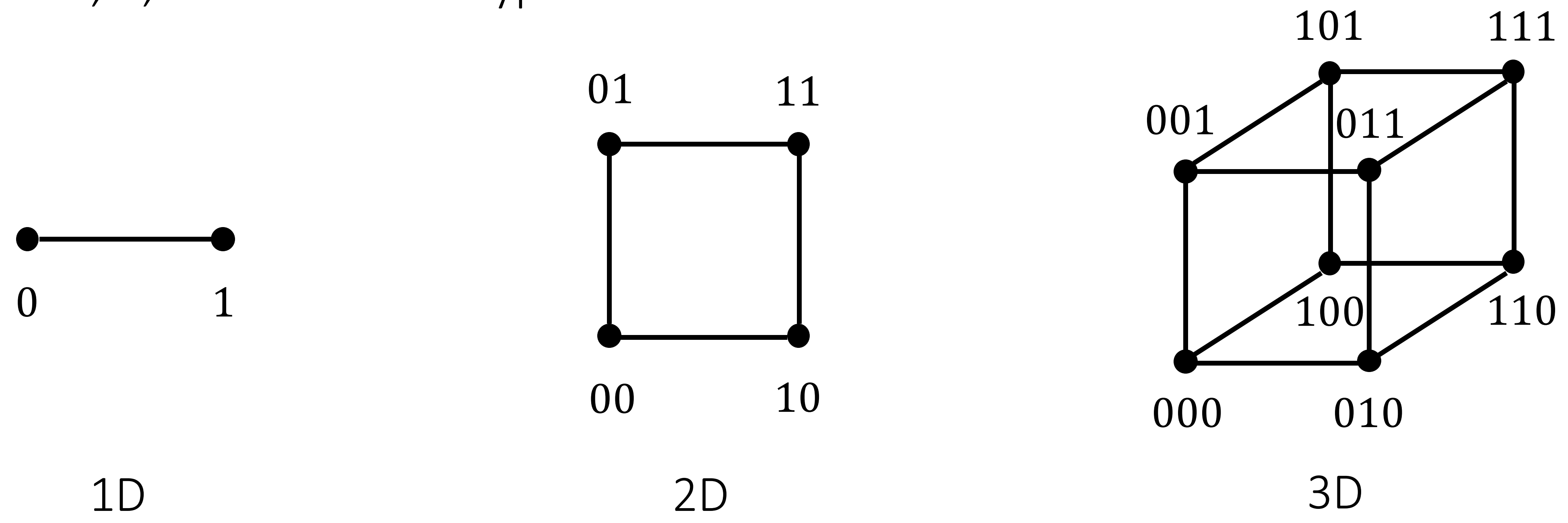
$$\Downarrow$$

$$v - e + f + \boxed{(k-1)} = 2k$$

Overcounted the inifinity face!

Problem 4: Hypercubes

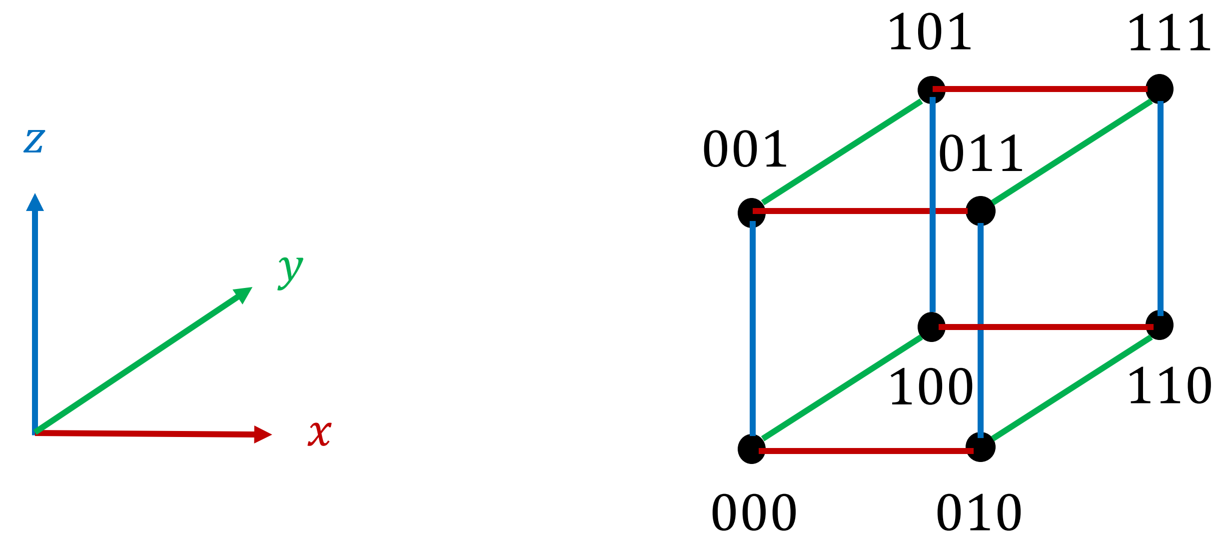
a) Draw 1, 2, 3 dimensional hypercubes



- Construction of hypercubes with bitstrings
- N dimensional hypercube = glueing two N-1 dimensional hypercube

Problem 4: Hypercubes

b) Show that the edges of an n-dimensional hypercube can be colored using n colors



Problem 4: Hypercubes

c) Show that n-dimensional hypercube is bipartite